Chapter 8: Introduction to Evolutionary Computation

Computational Intelligence: Second Edition

Contents

Some Theories about Evolution

- Evolution is an **optimization process**:
  - the aim is to improve the ability of an organism to **survive** in dynamically changing and competitive environments
- Two main theories on biological evolution:
  - Lamarckian (1744-1829) view
  - Darwinian (1809-1882) view

Lamarckian Evolution

- *Heredity*, i.e. the inheritance of acquired traits
- Individuals adapt during their lifetimes, and transmit their traits to their offspring
- Offspring continue to adapt
- Method of adaptation rests on the concept of use and disuse
- Over time individuals lose characteristics they do not require and develop those which are useful by "exercising" them

Darwinian Evolution

- Theory of **natural selection** and **survival of the fittest**
- In a world with limited resources and stable populations, each individual competes with others for survival
- Those individuals with the "best" characteristics (traits) are more likely to survive and to reproduce, and those characteristics will be passed on to their offspring
- These desirable characteristics are inherited by the following generations, and (over time) become dominant among the population
- During production of a child organism, random events cause random changes to the child organism's characteristics
- What about Alfred Wallace?

Evolutionary Computation

Evolutionary computation (EC) refers to computer-based problem solving systems that use computational models of evolutionary processes, such as

- natural selection,
- survival of the fittest, and
- reproduction,

as the fundamental components of such computational systems

Main Components of an Evolutionary Algorithm

- Evolution via natural selection of a randomly chosen population of individuals can be thought of as a search through the space of possible chromosome values
- In this sense, an evolutionary algorithm (EA) is a stochastic search for an optimal solution to a given problem
- Main components of an EA:
    - an **encoding** of solutions to the problem as a chromosome;
    - a **function** to evaluate the **fitness**, or survival strength of individuals;
    - **initialization** of the initial population;
    - **selection** operators; and
    - **reproduction** operators.

Generic Evolutionary Algorithm: Algorithm 8.1

Let $t = 0$ be the generation counter;
Create and initialize an n_x-dimensional population, $\mathcal{C}(0)$, to consist of n_s individuals;
while *stopping condition(s) not true* **do**
    Evaluate the fitness, $f(\mathbf{x}_i(t))$, of each individual, $\mathbf{x}_i(t)$;
    Perform reproduction to create offspring;
    Select the new population, $\mathcal{C}(t+1)$;
    Advance to the new generation, i.e. $t = t + 1$;
end

Evolutionary Computation Paradigms

- **Genetic algorithms** model genetic evolution.
- **Genetic programming**, based on genetic algorithms, but individuals are programs.
- **Evolutionary programming**, derived from the simulation of adaptive behavior in evolution (i.e. *phenotypic* evolution).
- **Evolution strategies**, geared toward modeling the strategic parameters that control variation in evolution, i.e. the evolution of evolution.
- **Differential evolution**, similar to genetic algorithms, differing in the reproduction mechanism used.
- **Cultural evolution**, which models the evolution of culture
- **Co-evolution**, in competition or cooperation

The Chromosome

- Characteristics of individuals are represented by long strings of information contained in the *chromosomes* of the organism
- Chromosomes are structures of compact intertwined molecules of DNA, found in the nucleus of organic cells
- Each chromosome contains a large number of genes, where a *gene* is the unit of heredity
- An alternative form of a gene is referred to as an *allele*

The "Artificial" Chromosome

In the context of EC, the following notation is used:

Chromosome (genome)	Candidate solution
Gene	A single variable to be optimized
Allelle	Assignment of a value to a variable

Two classes of evolutionary information:

- A *genotype* describes the genetic composition of an individual, as inherited from its parents
- A *phenotype* is the expressed behavioral traits of an individual in a specific environment

Binary Representation

- Binary-valued variables: $\mathbf{x}_i \in \{0,1\}^{n_x}$, i.e. each $x_{ij} \in \{0,1\}$
- Nominal-valued variables: each nominal value can be encoded as an n_d-dimensional bit vector where 2^{n_d} is the total number of discrete nominal values for that variable
- Continuous-valued variables: map the continuous search space to a binary-valued search space:
  - Each continuous-valued variable is mapped to an n_d-dimensional bit vector

$$\phi : \mathbb{R} \to (0,1)^{n_d}$$

Binary Representation (cont)

- Transform each individual

$$\mathbf{x} = (x_1, \ldots, x_j, \ldots, x_{n_x})$$

with $x_j \in \mathbb{R}$ to the binary-valued individual,

$$\mathbf{b} = (\mathbf{b}_1, \ldots, \mathbf{b}_j, \ldots, \mathbf{b}_{n_x})$$

where

$$\mathbf{b}_j = (b_{(j-1)n_d+1}, \ldots, b_{jn_d})$$

with $b_l \in \{0, 1\}$ and the total number of bits, $n_b = n_x n_d$

Binary Representation (cont)

- Decoding each $\mathbf{b}_j$ back to a floating-point representation:

$$\Phi_j(\mathbf{b}) = x_{min,j} + \frac{x_{max,j} - x_{min,j}}{2^{n_d} - 1}\left(\sum_{l=1}^{n_d-1} b_{j(n_d-l)} 2^l\right)$$

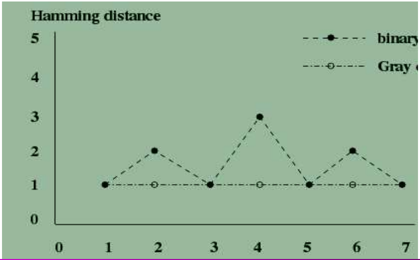
- That is, use the mapping,

$$\Phi_j : \{0,1\}^{n_d} \rightarrow [x_{min,j}, x_{max,j}]$$

Binary Representation (cont)

Problems with using a binary representation:

- Conversion form a floating-point value to a bitstring of n_d bits has the maximum attainable accuracy $\frac{x_{max,j}-x_{min,j}}{2^{n_d}-1}$ for each vector component, $j = 1, \ldots, n_x$
- Binary coding introduces Hamming cliffs:



- Hamming cliff is formed when two numerically adjacent values have bit representations that are far apart
- $7_{10} = 0111_2$ vs $8_{10} = 1000_2$
- Large change in solution is needed for small change in fitness

Gray Coding

- In Gray coding, the Hamming distance between the representation of successive numerical values is one
- For 3-bit representations:

	Binary	*Gray*
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

Converting binary strings to Gray bit strings:

$$
\begin{aligned}
g_1 &= b_1 \\
g_l &= b_{l-1}\overline{b}_l + \overline{b}_{l-1}b_l
\end{aligned}
$$

where b_l is bit l of the binary number

$$b_1 b_2 \cdots b_{n_b}$$

Gray Coding (cont)

A Gray code representation, $\mathbf{b}_j$ can be converted to a floating-point representation using

$$\Phi_j(\mathbf{b}) = x_{min,j} + \frac{x_{max,j} - x_{min,j}}{2^{n_d} - 1} \left(\sum_{l=1}^{n_d - 1} \left(\sum_{q=1}^{n_d - l} b_{(j-1)n_d + q} \right) \bmod 2 \right) 2^l$$

Other Representations

Other representations:

- Real-valued representations, where $x_{ij} \in \mathbb{R}$
- Integer-valued representations, where $x_{ij} \in \mathbb{Z}$
- Discrete-valued representations, where $x_{ij} \in \text{dom}(x_j)$
- Tree-based representations as used in Genetic Programming
- Mixed representations

Initialization

- A population of candidate solutions is maintained
- Values are randomly assigned to each gene from the domain of the corresponding variable
- Uniform random initialization is used to ensure that the initial population is a uniform representation of the entire search space
- What are the consequences of the size of the initial population?
  - Computational complexity per generation
  - Number of generations to converge
  - Quality of solutions obtained – Exploration ability

Survival of the Fittest

- Based on the Darwinian model, individuals with the best characteristics have the best chance to survive and to reproduce
- A *fitness function* is used to determine the ability of an individual of an EA to survive
- The fitness function, f, maps a chromosome representation into a scalar value:

$$f : \Gamma^{n_x} \rightarrow \mathbb{R}$$

  where Γ represents the data type of the elements of an n_x-dimensional chromosome

Objective Function

- The fitness function represents the objective function, Ψ, which represents the optimization problem
- Sometimes the chromosome representation does not correspond to the representation expected by the objective function, in which case

$$f : \mathcal{S}_C \xrightarrow{\Phi} \mathcal{S}_X \xrightarrow{\Psi} \mathbb{R} \xrightarrow{\Upsilon} \mathbb{R}_+$$

  $\mathcal{S}_C$ represents the search space of the objective function
  Φ represents the chromosome decoding function
  Ψ represents the objective function
  Υ represents a scaling function
- For example:

$$f : \{0,1\}^{n_b} \xrightarrow{\Phi} \mathbb{R}^{n_x} \xrightarrow{\Psi} \mathbb{R} \xrightarrow{\Upsilon} \mathbb{R}_+$$

Objective Function Types

Types of objective functions, resulting in different problem types:

- **Unconstrained** objective functions, but still subject to boundary constraints
- **Constrained** objective functions
- **Multi-objective** functions, where more than one objective has to be optimized
- **Dynamic** or **noisy** objective functions

Selection Operators

- Relates directly to the concept of survival of the fittest
- The main objective of selection operators is to emphasize better solutions
- Selection steps in an EA:
  - Selection of the new population
  - Selection of parents during reproduction
- Selective pressure:
  - Takeover time
  - Relates to the time it requires to produce a uniform population
  - Defined as the speed at which the best solution will occupy the entire population by repeated application of the selection operator alone
  - High selective pressure reduces population diversity

Random Selection

- Each individual has a probability of $\frac{1}{n_s}$ to be selected
- n_s is the size of the population
- No fitness information is used
- Random selection has the lowest selective pressure

Proportional Selection

- Biases selection towards the most-fit individuals
- A probability distribution proportional to the fitness is created, and individuals are selected by sampling the distribution,

$$\varphi_s(\mathbf{x}_i(t)) = \frac{f_\Upsilon(\mathbf{x}_i(t))}{\sum_{l=1}^{n_s} f_\Upsilon(\mathbf{x}_l(t))}$$

n_s is the total number of individuals in the population
$\varphi_s(\mathbf{x}_i)$ is the probability that $\mathbf{x}_i$ will be selected
$f_\Upsilon(\mathbf{x}_i)$ is the scaled fitness of $\mathbf{x}_i$, to produce a positive floating-point value

Proportional Selection: Sampling Methods

- Roulette wheel selection: Assuming maximization and normalized fitness values (Algorithm 8.2):

Let $i = 1$, where i denotes the chromosome index;
Calculate $\varphi_s(\mathbf{x}_i)$;
$sum = \varphi_s(\mathbf{x}_i)$;
Choose $r \sim U(0,1)$;
while $sum < r$ **do**
    $i = i + 1$, i.e. advance to the next chromosome;
    $sum = sum + \varphi_s(\mathbf{x}_i)$;
end
Return $\mathbf{x}_i$ as the selected individual;

- High selective pressure

Proportional Selection: Stochastic Universal Sampling

- Uses a single random value to sample all of the solutions by choosing them at evenly spaced intervals
- Like roulette wheel sampling, construct a wheel with sections for each of the n_s chromosomes
- Instead of one hand that is spun once for each sample, a multi-hand is spunt just once
- The hand has n_s arms, equally spaced
- The number of times chromosome i is sampled is the number of arms that fall into the chromosomes section of the wheel

Tournament Selection

- Selects a group of n_{ts} individuals randomly from the population, where $n_{ts} < n_s$
- Select the best individual from this tournament
- For crossover with two parents, tournament selection is done twice
- Tournament selection limits the chance of the best individual to dominate
- Large n_{ts} versus small n_{ts}
- Selective pressure depends on the value of n_{ts}

Rank-Based Selection

- Uses the rank ordering of fitness values to determine the probability of selection
- Selection is independent of absolute fitness
- Non-deterministic linear sampling:
  - Selects an individual, $\mathbf{x}_i$, such that $i \sim U(0, U(0, n_s - 1))$
  - The individuals are sorted in decreasing order of fitness value
  - Rank 0 is the best individual
  - Rank $n_s - 1$ is the worst individual

Rank-Based Selection (cont)

- Linear ranking:
  - Assumes that the best individual creates $\hat{\lambda}$ offspring, and the worst individual $\tilde{\lambda}$
  - $1 \leq \hat{\lambda} \leq 2$ and $\tilde{\lambda} = 2 - \hat{\lambda}$
  - The selection probability of each individual is calculated as

$$\varphi_s(\mathbf{x}_i(t)) = \frac{\tilde{\lambda} + (f_r(\mathbf{x}_i(t))/(n_s - 1))(\hat{\lambda} - \tilde{\lambda})}{n_s}$$

  where $f_r(\mathbf{x}_i(t))$ is the rank of $\mathbf{x}_i(t)$

Rank-Based Selection (cont)

- Nonlinear ranking:

$$\varphi_s(\mathbf{x}_i(t)) = \frac{1 - e^{-f_r(\mathbf{x}_i(t))}}{\beta}$$

$$\varphi_s(\mathbf{x}_i) = \nu(1-\nu)^{n_p - 1 - f_r(\mathbf{x}_i)}$$

  $f_r(\mathbf{x}_i)$ is the rank of $\mathbf{x}_i$
  β is a normalization constant
  ν indicates the probability of selecting the next individual
- Rank-based selection operators may use any sampling method to select individuals

Boltzmann Selection

- Based on the thermodynamical principles of simulated annealing
- Selection probability:

$$\varphi(\mathbf{x}_i(t)) = \frac{1}{1 + e^{f(\mathbf{x}_i(t))/T(t)}}$$

$T(t)$ is the temperature parameter

- An initial large value ensures that all individuals have an equal probability of being selected
- As $T(t)$ becomes smaller, selection focuses more on the good individuals
- Can use any sampling method

Boltzmann Selection (cont)

- Boltzmann selection can be used to select between two individuals
- For example, if

$$U(0,1) > \frac{1}{1 + e^{(f(\mathbf{x}_i(t)) - f(\mathbf{x}'_i(t)))/T(t)}}$$

then $\mathbf{x}'_i(t)$ is selected; otherwise, $\mathbf{x}_i(t)$ is selected

$(\mu \overset{+}{,} \lambda)$-Selection

- Deterministic rank-based selection methods used in evolutionary strategies
- μ indicates the number of parents
- λ is the number of offspring produced from each parent
- (μ, λ)-selection selects the best μ offspring for the next population
- $(\mu + \lambda)$-selection selects the best μ individuals from both the parents and the offspring

Elitism

- Ensures that the best individuals of the current population survive to the next generation
- The best individuals are copied to the new population without being mutated
- The more individuals that survive to the next generation, the less the diversity of the new population

Hall of Fame

- Similar to the list of best players of an arcade game
- For each generation, the best individual is selected to be inserted into the hall of fame
- Contain an archive of the best individuals found from the first generation
- The hall of fame can be used as a parent pool for the crossover operator

Types of Operators

- **Reproduction**: process of producing offspring from selected parents by applying crossover and/or mutation operators
- **Crossover** is the process of creating one or more new individuals through the combination of genetic material randomly selected from two or more parents
- **Mutation**:
  - The process of randomly changing the values of genes in a chromosome
  - The main objective is to introduce new genetic material into the population, thereby increasing genetic diversity
  - Mutation probability and step sizes should be small
  - Proportional to the fitness of the individual?
  - Start with large mutation probability, decreased over time?

When to Stop

- Limit the number of generations, or fitness evaluations
- Stop when population has converged:
  - Terminate when no improvement is observed over a number of consecutive generations
  - Terminate when there is no change in the population
  - Terminate when an acceptable solution has been found
  - Terminate when the objective function slope is approximately zero

EC vs CO

- The search process:
  - CO uses deterministic rules to move from one point in the search space to the next point
  - EC uses probabilistic transition rules
  - EC applies a parallel search of the search space, while CO uses a sequential search
- Search surface information:
  - CO uses derivative information, usually first-order or second-order, of the search space to guide the path to the optimum
  - EC uses no derivative information, but fitness information